Temporal Dissagregation using the State Space Approach

Discussion paper 05004

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Explanation of symbols

.	= data not available
*	= provisional figure
x	= publication prohibited (confidential figure)
–	= nil or less than half of unit concerned
–	= (between two figures) inclusive
0 (0,0)	= less than half of unit concerned
blank	= not applicable
2004–2005	= 2004 to 2005 inclusive
2004/2005	= average of 2004 up to and including 2005
2004/'05	= crop year, financial year, school year etc. beginning in 2004 and ending in 2005

Due to rounding, some totals may not correspond with the sum of the separate figures.

Statistics Netherlands

TEMPORAL DISAGGREGATION USING THE STATE SPACE APPROACH

Summary: In this paper, the state space approach is used to perform temporal disaggregation. The investigation is performed in two steps. First two study cases are used to gain information about the reliability and the accuracy of the approach, while, in the second step, the method is applied to the retail sales statistics in order to obtain estimates for the monthly turnover sales of the retail sector from January 1994 to December 1999.

In the first test case, Monte-Carlo simulations are used to investigate the ability of the state space method to reproduce the data generating process of the disaggregated series which is, in that case, a combination of a linear trend and an autoregressive process of order 1. From the results, it is seen that only a few observations are needed to reproduce the slope accurately while the representation of the autoregressive process requires more observations. Further, it is clear from this test case that the use of the filtered series (obtained from the state space approach) reproduces the original series more accurately than simulations.

From the second test case, based on the monthly retail sales statistics, it appears that the representation of the trend has a large influence on the description of the disaggregated series, the model with the largest flexibility providing the best representation of the original series. Looking closer at the yearly data (i.e. the aggregated series), it appears that the inclusion of initial conditions leads to discrepancies between the original yearly data and the sum of the monthly data. The use of Denton's method is found to provide an elegant way to distribute the differences among the monthly data. Comparing Denton's method to more naïve approaches nicely illustrates the advantages of Denton's algorithm.

Finally, the combination of the state space approach with Denton's method is used to disaggregate the data from the yearly statistics on retail sales. This is needed if one wants to estimate the monthly sales from the retail sector because the monthly statistics only rely on relative levels. In this application, the positive influence of Denton's algorithm on the representation of the series is clearly illustrated.

The conclusion of the present paper is that the combination of the two methods provides a powerful tool to disaggregate annual data in higher frequency series.

Keywords: Temporal disaggregation, Time series, State Space approach, Denton's adjustment method, Retail sales statistics.

1. Introduction

A problem often faced in macroeconomics is the interpolation or the distribution of economic time series observed at low frequency into compatible higher frequency data. Such a process, called temporal disaggregation, plays an important role in the estimation of short-term indicators and several national statistical institutes of European countries make extensive use of these techniques[1], for example when a set of indicators available at the quarterly frequency is used for constructing the quarterly national economic account from annual figures.

In the Eurostat handbook, a distinction between direct and indirect disaggregation approaches is made. Direct procedures are based on the availability of similar sources as those used to compile the annual accounts at higher frequency intervals, with appropriate simplifications. On the other hand, indirect procedures are based on the time disaggregation of the annual accounts data in accordance with mathematical and statistical methods using reference indicators that permit the extrapolation for the current year. In addition, several methods were developed for the cases where no information on the higher frequency structure is available. The estimates are then obtained from assumptions about the relation between the data. These purely mathematical approaches are the only methods that can be used when there are serious gaps in the basic information, where the only data available are those pertaining to the annual series.

In order to help member states to perform temporal disaggregation, Eurostat has developed the Ecotrim software tool[2], which supplies procedures based on temporal disaggregation of low frequency time series via mathematical and statistical methods. Although Ecotrim is a useful package, the code does not offer the possibility to use the very powerful state space approach.

The use of the state space approach for performing temporal disaggregation was first introduced by Harvey and Pierce in 1984 and later developed by, among others, Durbin and Quenneville (1997), Harvey (1989), Harvey and Chung (2000), Harvey and Koopman (1997) and Moauro and Savio (2002) who proposed an approach based on the use of seemingly unrelated structural time series to deal with multivariate time series. Here, the study will be restricted to univariate series but the extension to multivariate systems is straightforward.

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[1] See the "Handbook on quarterly accounts", published by Eurostat.
[2] The Ecotrim software is available on request at Eurostat. For a description of the program, we refer to the user manual (Barcellan and Buono, 2002), available at the Eurostat website.

In the present work, we first briefly introduce the theoretical approach (in section 2). Then, in section 3, two case studies are presented. In the first one, a DGP[3] is used to produce a monthly time series which is aggregated to year data. The yearly series is then disaggregated using the state space approach and a comparison between the original and the computed series is used to investigate the accuracy of the decomposition. In the second test case, a "semi-practical" case is treated. The monthly index series from the statistics on retail sales (published by Statistics Netherlands) is used to determine the seasonal component that will be used further in chapters 3 and 4. In addition the disaggregation is investigated by performing calculations similar to these reported for the first case study. In section 4, an application, similar to the case studies, is reported. The data of the yearly retail sales statistics are disaggregated using a seasonal component extracted from the monthly statistic. This approach allows to estimate the monthly sales of the retail sector, while the monthly statistic is built on relative levels. Finally conclusions are presented in section 5.

2. Theoretical background

2.1 State space approach[4]

The state space model for univariate or multivariate time series is based on a set of two definition equations. The first one, called observation or measurement equation, expresses the observation vector, Y, as a linear function of a state variable, X, plus noise.

(1) $Y_t = G_t X_t + W_t, \qquad t = 1,....,n$

where W is white noise and G is a sequence of (not necessarily square) matrices. The second equation, named state or transition equation, defines the state variable X_{t+1} in terms of the previous state, X_t, plus a noise term.

(2) $X_{t+1} = F_t X_t + V_t, \qquad t = 2,...,n$

where V_t is a white noise and F is a sequence of matrices. The conditions under which the state space approach can be used are that V and W must be uncorrelated at all times and that the initial state X_1 is uncorrelated with all the noise terms (V and W).

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[3] Data Generating Process.
[4] In this section, only a short description of the method is reported. For detailed information on the method or a description of the Kalman filter, we refer to Brockwell and Davis (2002) or Durbin and Koopman (2001).

In the present case, it is useful to decompose the two equations in such a way that the influence of the exogenous regressors becomes clearly visible. The observation equation and the state equation can then be written as:

$$(3) \quad Y_t = Z'\alpha_t' + x_t'\beta + Q\varsigma_t, \quad t = 1,....,n$$

and

$$(4) \quad \alpha_t = T_{t-1}\alpha_{t-1} + W_{t-1}\beta + H_{t-1}\varepsilon_{t-1}, \quad t = 2,.....,n$$

Noticing that the state equation depends on the initial condition (i.e. α_1, T_1, W_1 and H_1) and that the two errors terms (ε and ζ) are normally distributed. The vectors x_t and the matrices W_t contain exogenous regressors that enter respectively the measurement equation and the transition equation and zero elements corresponding to effects that are absent from one or the other equations.

Assuming that the Y_t's are not observed but that information about temporally aggregated series, $\Sigma_{j=0, ..., s-1} Y_{\tau s - j}$, is available at times $\tau = 1, 2, \ldots [n/s]$ [5], it is possible to apply the approach proposed by Harvey (1989) to obtain a representation of the disaggregated series. This approach is based on the definition of the so-called cumulator as

$$(5) \quad A_t = \Psi_t A_{t-1} + Y_t$$

ψ_t is a function always equal to one, except when $t = s(\tau - 1) + 1$, where τ goes from 1 to $[n/s]$. In that case ψ_t is equal to 0. The state space representation for A_t can be written in terms of $\alpha_t^* = [\alpha_t, A_t]$' as

$$(6) \quad A_t = Z^{*'}\alpha_t^{*'}; Z^* = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and

$$(7) \quad \alpha_t^* = T_{t-1}^*\alpha_{t-1}^* + W_{t-1}^*\beta + H_{t-1}^*\varepsilon_{t-1}, \quad t = 2,.....,n$$

Noticing that the regression effects are all contained in the state equation and that the noise in the measurement equation was dropped. Moreover, the transition matrix, T^*, is time varying because it includes ψ_t.

From these equations, the A_t's can be estimated using the state space approach, while the disaggregated series is obtained by reversing the definition equation of the cumulator.

The flexibility of the approach allows to include AR terms in the definition equations and, in the present case, an AR(1) process is included via the T

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[5] Where $[p]$ denotes the integral part of p.

matrix. The representation of a series by a linear model with first order auto-regressive errors was first introduced by Chow and Lin in 1971.

2.2 Denton's adjustment method

Denton (1971) developed an algorithm allowing to distribute the discrepancy between the sum of the monthly data and the related yearly results. The approach is based on the definition of a so-called penalty function. In the present work, the penalty function to be minimized, p, is based on the differences between the first differences of the original, z, and adjusted, x, series.

$$(8) \quad p(x,z) = \sum_{t=1}^{n} (\Delta x_t - \Delta z_t)^2$$

where Δ is the backward difference operator[6]. The correction matrix is computed in several steps. Here, the different steps of the process will not be repeated and for more information on the theoretical background, we refer to the original paper (Denton 1971). The main steps of the process are, first, the use of Lagrange multipliers and, secondly, the computation of the inverse of a partitioned matrix. The final result is that the adjusted and the original series are related by the following equation

$(9) \ x = z + A^{-1} B (B' A^{-1} B)^{-1} (y - B'z)$

where y is a vector containing the yearly totals and B is a diagonal square matrix of vectors linking the monthly data to their annual sum. The matrix A is built on the penalty function by first expressing the vector of backwards first differences as $D(x-z)$, where D is an n by n matrix and $(x-z)$ an n by 1 vector. The quadratic form to be minimized can now be written as $(x-z)'D'D(x-z)$ or, in the present notation, $(x-z)'A(x-z)$.

3. Case studies

3.1 Stationary process around a deterministic trend

The state space approach is partly based on the Kalman filter (Kalman 1960). This powerful technique uses recurrence relations to gain information on the system, which involves that the accuracy of the state space description is expected to improve with t, $t = 1, \ldots, n$; n being the number of observations (yearly data in the present case). The first case under study is a stationary
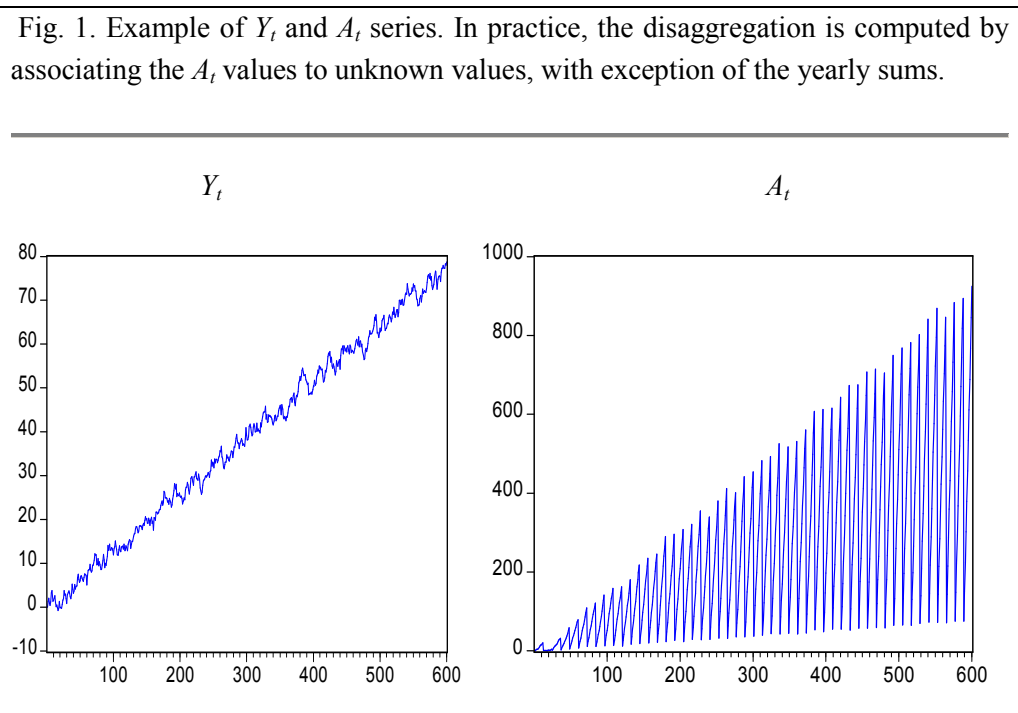
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[6] $\Delta x_t = x_t - x_{t-1}$.

process around a deterministic trend. The stationary process is represented by an AR(1) model following

$$Y_t = 0.765Y_{t-1} + W_t, \qquad W \sim NID(0,1)$$

while the deterministic trend has a slope of 0.1293. The aggregated series was obtained by summing the Y_t, considering that the DGP is associated with monthly data.

The tests were performed on 4 series of different lengths, containing respectively 72, 144, 300 and 600 monthly data. This involves that the state space representation relies on, respectively, 6, 12, 25 and 50 "valid" observations (i.e. the yearly results). An example of the DGP is shown in fig. 1, together with the complete A_t function.

Fig. 1. Example of Y_t and A_t series. In practice, the disaggregation is computed by associating the A_t values to unknown values, with exception of the yearly sums.

The accuracy of the state space description is investigated by Monte-Carlo simulations using 2500 runs. For each of the 4 series, the mean, the median and the standard deviations are reported in table 1.

In the DGP, the slope is not perturbed by any noise. Therefore it is expected that the accuracy of the representation will improve with increasing number of observations. It is shown in table 1 that both the mean and the median of the Monte-Carlo simulations rapidly converge to the DGP value. The representation based on only 6 observations already gives an accurate description of the slope, the mean being less than 1 % off the DGP value. Using 12 observations reduces the error by a factor of 10 while further increase has only marginal influence on the accuracy. The standard deviation shows a much

stronger dependence on the number of observations than the mean and the median. As expected, the standard deviation decreases with increasing number of observations and, from the results presented in table 1, it looks like the decrease follows an exponential curve.

Table1: Results of the Monte-Carlo simulations for the 4 series (see text). For each series, characterized by the number of observations, n, the mean, the median and the standard deviation of the two coefficients are reported. The coefficients are related to the trend and the AR process.

	Trend			AR coefficient		
n	Mean	Med	Std. dev.	Mean	Med	Std. dev
6	0.1300	0.1295	0.0381	0.7486	0.7627	0.1308
12	0.1292	0.1294	0.0080	0.7937	0.7907	0.0738
25	0.1293	0.1293	0.0016	0.7842	0.7945	0.0919
50	0.1293	0.1293	0.0005	0.7592	0.7952	0.1840
DGP	0.1293			0.7650		

The determination of the AR(1) coefficient behaves differently. Because the generation of the AR(1) process involves normally distributed random series, it is expected that the accuracy of the method will be lower than for the slope. The results for the different Monte-Carlo simulations are reported in table 1. Using 6 observations leads to an underestimation of the mean, while the median is lying very close to the DGP value. The standard deviation, 0.13, is rather large (a factor 7/2 larger than for the slope) which explains the large difference between the mean and the median, 0.015.

Increasing the number of observations by a factor of 2 leads to a decrease of the standard deviation by a factor of 2 but, in that case, both the mean and the median are further from the original value than for $n = 6$. A further increase of the number of observations to 25 does not solve the problem. The mean and the median remain centred on 0.79 while the standard deviation in-

creases, which is unexpected. A further increase of the number of observations, up to 50, is needed to have the mean approaching the DGP value by 0.8 %. Surprisingly, the standard deviation is larger than for $n = 6$, 12 or 25, which is also illustrated by the large difference between the mean and the median, 0.035. This means that, if a single run is taken for simulating the series, large differences can be observed.

To investigate further the behaviour of the system, an extra Monte-Carlo simulation on basis of 100 observations was performed, focussing on the behaviour of the standard deviation. In that case, the observed mean is 0.746, which is slightly further from the original value than for $n = 50$ but the standard deviation is reduced to 0.146, being 20 % smaller than for $n = 50$.

This example clearly illustrates the bottleneck of the method. In order to achieve accurate results, observations over 50 years or more are needed.

Fig. 2. Comparison between the original series (DGP) and a simulation (SIMUL) based on one of the runs (chosen at random) of the Monte-Carlo approach (for $n = 50$). In fig. 2II, only the last 100 points are shown which allows to give a better representation of the short-time differences between the two series
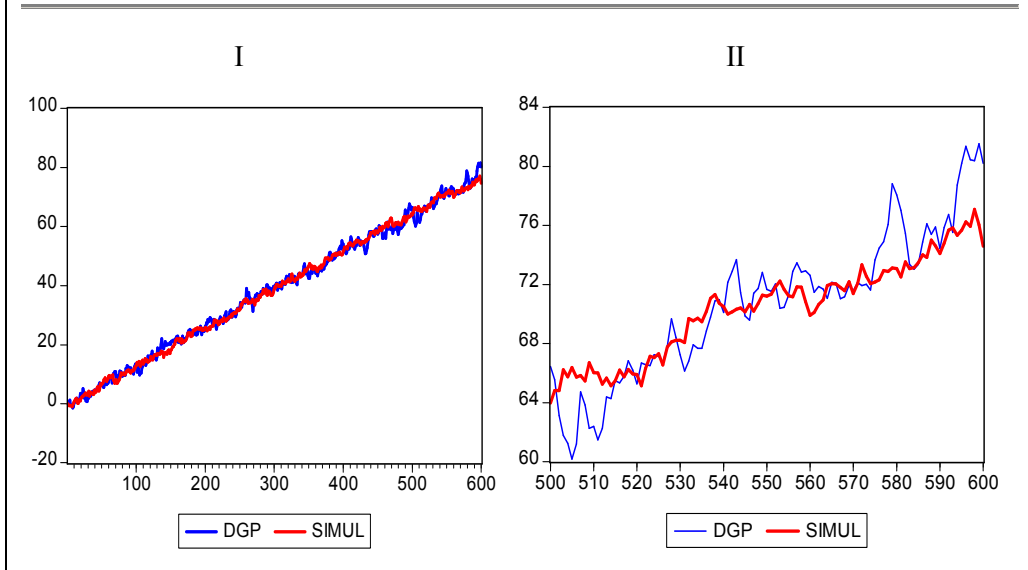

Results for a single run, taken at random, are shown in fig. 2. From fig. 2I, it is seen that the simulation has the right long-term behaviour but fig. 2II shows that, because random series are involved, differences in the short-term behaviour are observed.

However, the main goal of the approach presented here is not to retrieve the DGP but to give an accurate representation of the original series. From the results presented in table 1, it is clear that a simulation on basis of the Monte-Carlo results leads to accurate long term behaviour (shown in fig. 2I) while

the representation of the short-time (see fig. 2II) is less accurate. In addition, the condition that the yearly sum of the simulated data should be equal to the observed values is nearly never fulfilled. Therefore we will focus on the filtered series which is provided by the state space approach.

Fig. 3. Comparison between the original (DGP) and filtered (FILTER) series. In fig. 2II, the series are detrended in order to give a better representation of the short-time differences.
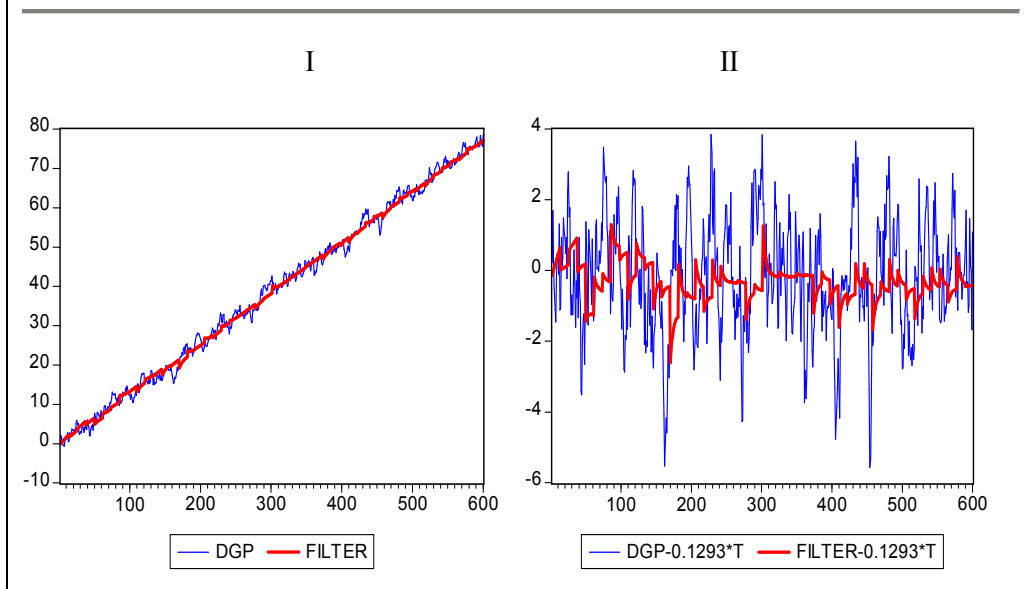

A comparison between the filtered and original series is shown in fig. 3. As expected, the long-term behaviour is well described (fig. 3I) while differences in the short-time behaviour (fig. 3II) are more difficult to analyse, the amplitude of the detrended original series being about a factor 4 larger than for the filtered series.

Fig. 4 provides a better tool for analysing the accuracy of the state space description. In that figure, the results of a simulation based on 2500 runs are reported. For each run, the differences between the original and filtered yearly results are computed. The series called "DIFF" represents the mean distance[7]. As expected, the method gains information about the system with increasing time (i.e. the error decreases with increasing t). This effect is even better illustrated by the "RDIFF" series in which the mean of the relative distance is reported. From fig. 4, it appears that the chance for erratic description is much larger for the 15 first observations than for the rest.

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[7] In this case, the distance is defined as the absolute value of the difference between the original and the filtered series.

The use of Denton's technique to spread the differences between the series smoothly among the monthly data will be discussed in sections 3.2 and 4.

Fig. 4. Distances between the original and filtered data for the yearly observations. The series "DIFF" and "RDIFF" correspond respectively to the mean absolute and the mean relative distances over a series of 2500 simulations (see text).
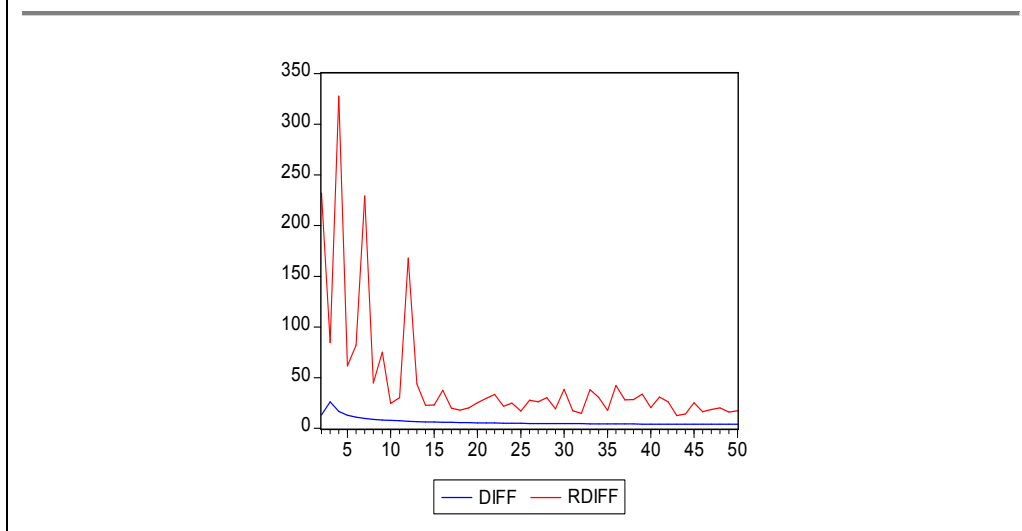
### 3.2 Test on the data from the retail sales statistics

The second test case is built on the data of the monthly retail sales statistics. The observations are available for a period of about ten years, going from January 1994 to October 2004[8]. Using a deterministic approach, the data can be fitted (using a linear regression) as the sum of a seasonal pattern (modelled using dummy variables) and a trend polynomial of degree 3[9]. The results of the regression are not reported here but the original and the fitted series are presented in fig. 5, together with the residuals. From this regression, the deterministic seasonal behaviour is extracted in order to create an artificial seasonal indicator which will be used as variable in the disaggregation (i.e. it is part of the $W^*\beta$ in the state equation (cf. equation (7)). In a second step the monthly data are summed to obtain the yearly results and the disaggregation is applied to the yearly series. The reason why the seasonal pattern is treated as a variable is that in practical cases external information about the seasonal behaviour is required because this kind of information is not contained in the yearly data.

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[8] The data are not related to absolute values but to relative levels. The reference is the year 1995 that has a mean value of 100.
[9] i.e. $a_1\, t + a_2\, t^2 + a_3\, t^3$

From the results of the deterministic regression, it is clear that, although they are small, respectively 0.015 and -0.00008, the coefficients of t^2 and t^3 are highly significant. The values of the individual t-statistics being respectively 10.6 and -11.7. However, it was decided not to include these two terms as exogenous variables in the state equation. This allows to investigate if the AR(1) process can compensate for the missing information (i.e. the small quadratic and cubic effects are approximated by an AR(1) process). The motivation is that the yearly data, which are, together with the seasonal pattern, the only information provided to the system, seems to follow a quasi-linear trend.

Fig. 5. Original and fitted series for the monthly retail sales. The fitted data are obtained by performing a linear regression including monthly dummies and a trend polynomial of degree 3 as regressors. The data presented here are not related to absolute values but to relative levels. The reference is the year 1995 that has a mean value of 100. The residuals of the regression are also presented.
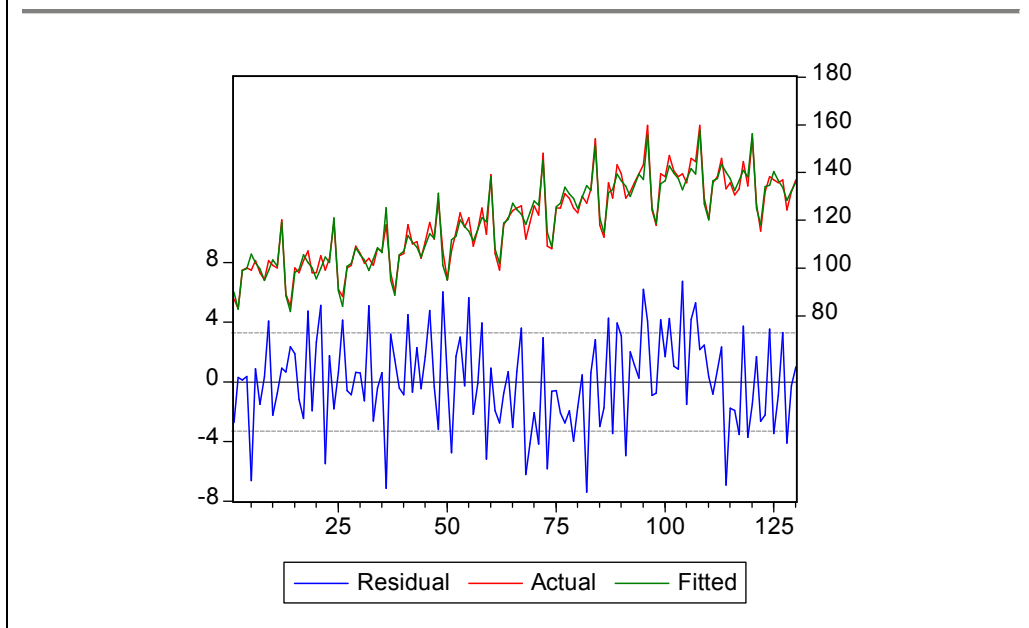
Three different state space models were tested. In the three cases, the AR(1) process is described by the same equation which involves that the models only differ by the description of the trend. In model A, the coefficient of the trend is a constant, while in model B the trend coefficient is allowed to vary smoothly. The latter is expected to give a better representation of the system around the turning points, observed close to t = 35 and 110 (see fig. 5). In model C, a stochastic trend is introduced in the state equation. The results are

expected to be close to these obtained for model B, but here the trend is not an exogenous variable anymore[10].

Before comparing the accuracy of the representations, it is interesting to look at the results presented in table 2. In this table, the coefficients of the seasonal pattern and the coefficients of the AR(1) process are reported. Although some differences are observed, the three coefficients of the seasonal pattern are close to 1, which gives a good representation of the original process.

Table 2: Coefficients of the seasonal pattern, the AR(1) process and the trend for models A, B and C (see text). In addition, the results of a linear regression (including the same regressors) are added for comparison.

| Model	Seasonal pattern	AR(1)	Trend
A	1.008	1.000	0.3233
B	0.980	-0.18	*
C	0.974	0.00002	*
OLS	0.956	0.537	0.3654

* Because in models B and C the coefficient of the trend is allowed for vary, it can not be reduced to a single number.

The differences for the AR coefficients are much larger. From table 2 it is seen that model A compensates the lack of information by representing the system as the sum of a seasonal pattern and a random walk. Comparing this to the results obtained from a linear regression (with a linear trend and an AR(1) process as regressors), it appears that the AR process is about a factor of two larger when using the state space approach. The inclusion of some flexibility in the trend (for model B and C) involves that the AR(1) process is not the only process compensating for the missing information anymore. This is the reason why the AR coefficient in model B is a factor of 5 smaller than

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[10] The trend and the seasonal component are part of the $W^*\beta$ term from equation (7). In the three models, the seasonal component is then seen as a variable.

for model A, while the higher flexibility of the trend in model C makes the AR(1) process irrelevant.

Fig. 6. Graphical representation of the state space analysis of the retails sales series. In figures I, IV and VII, the filtered series are presented, while in figures II, V and VII, the differences between the filtered and the original series are shown. In figures III, VI and IX, the differences between the original and filtered yearly results are reported. Figures I, II and III are associated with model A, figures IV, V and VI are associated with model B and figures VII, VIII and IX are associated with model C.
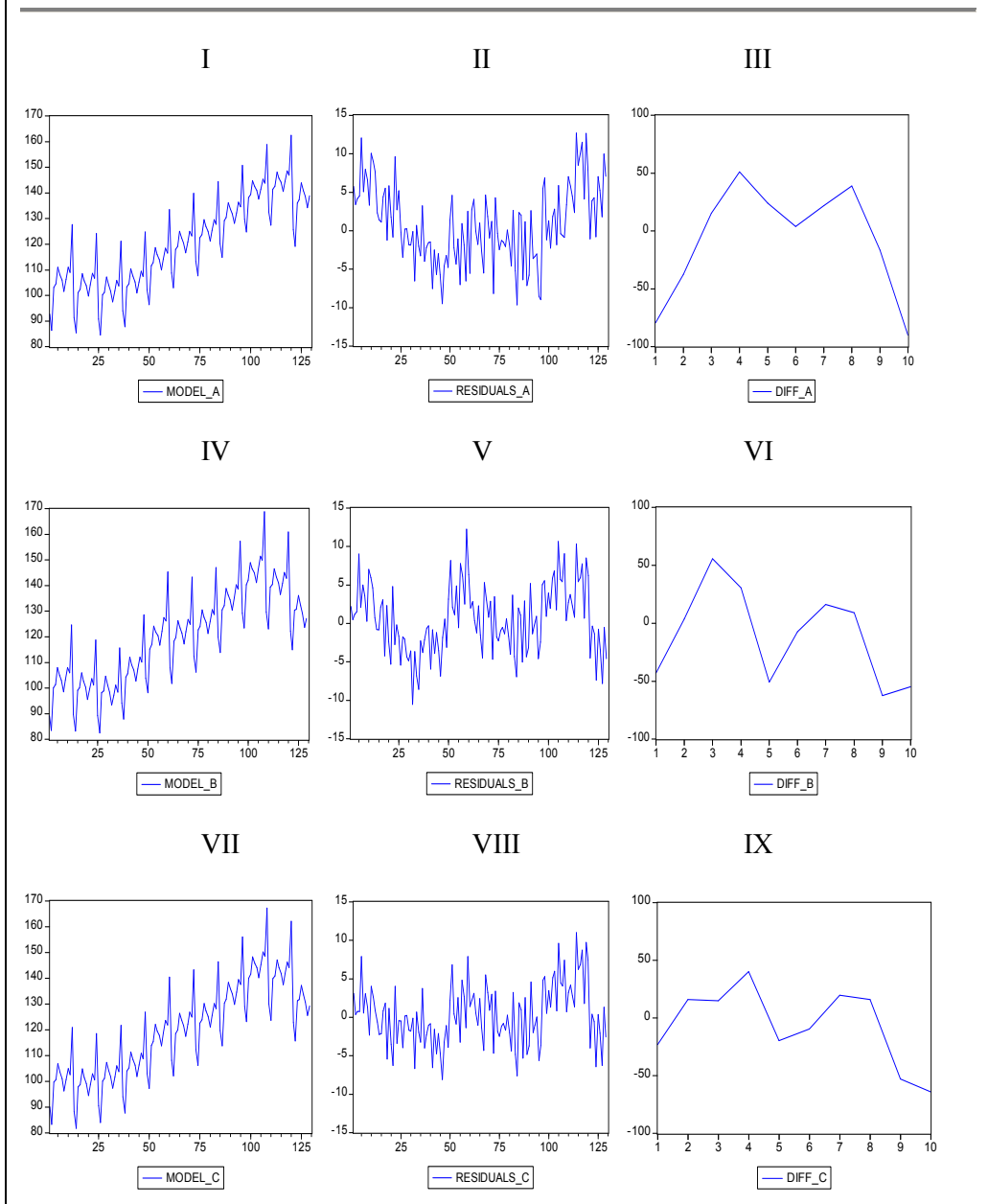
The results are graphically represented in fig. 6 and fig. 7. In fig. 6 the filtered series are plotted (fig. I, IV and VII) together with the monthly (fig. II, V and VIII) and yearly residuals (fig. III, VI and IX). The monthly residuals are obtained by subtracting the original monthly data from the filtered series. The yearly residuals are obtained in a similar way, the data being previously summed to obtain the yearly observations. The differences between the filtered series are better represented in fig. 7 where the differences between model A and model C (fig. 7I) and between model B and model C (fig. 7II) are shown.

Fig. 7. Differences between the filtered series. In fig. 7I, the differences between the representations of model A and model C are plotted, while fig. 7II, shows the differences between model B and C.
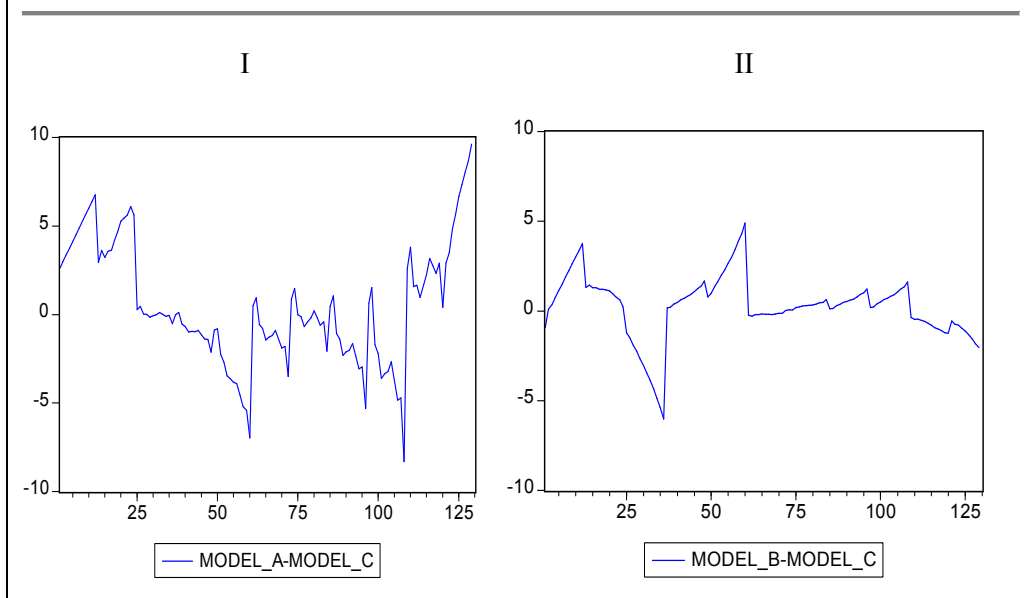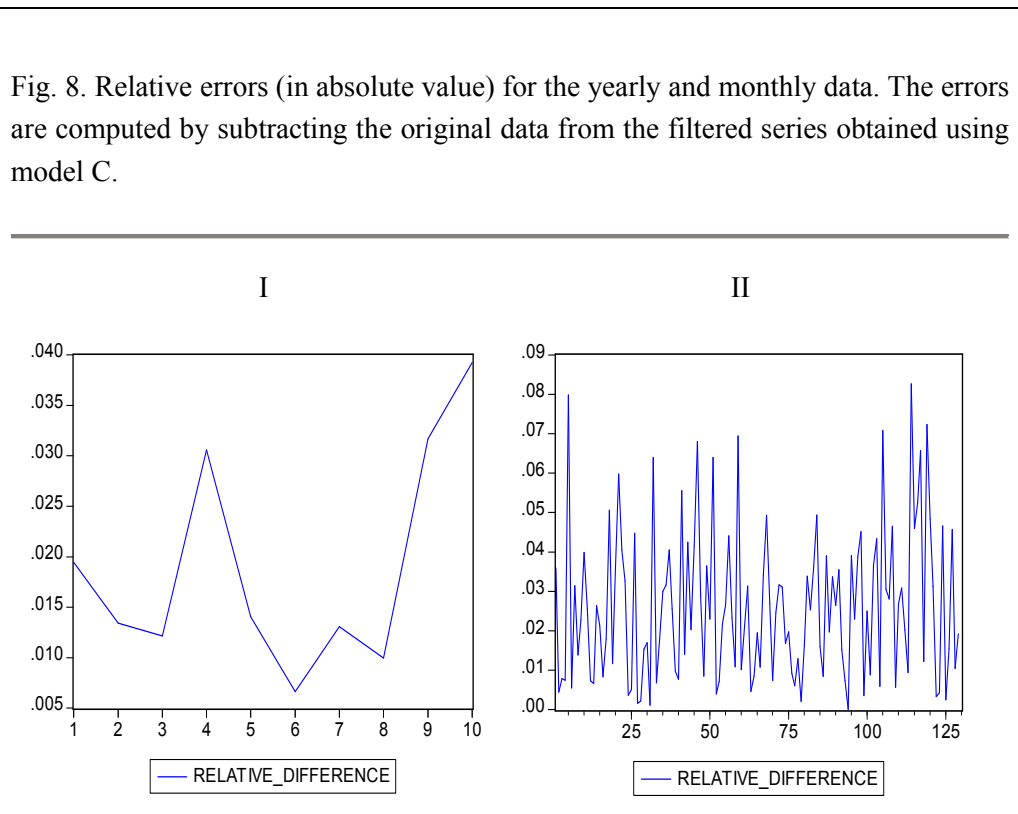

Looking closely at the filtered series reported in fig. 6, it seems that the series show different behaviours close to the turning points ($t \sim 35$ and $t \sim 110$). These differences are better illustrated in fig. 7. In fig 7I, the largest differences are found for $t < 25$ and $t > 100$. The two first peaks could be explained by the adaptation time needed by model A to compensate for the rigidity of the description (i.e. the lack of flexibility in the trend). This assumption is supported by fig. 6III and 6IX, where the distances to the original data are reported. From these two graphs, it is seen that model A poorly reproduces the first two yearly data while the description by model C is more accurate. The same can be concluded on basis of fig. 6II and fig. 6VIII. The difference between model B and C around the first turning point is clearly illustrated by the peak around $t = 35$ in fig. 7II. In addition, the poor representation of the series by model B around $t = 60$ is clearly visible from

tation of the series by model B around t = 60 is clearly visible from fig. 6IV, V and VI. It corresponds to a jump in the filtered series caused by a rescaling of the trend.

From fig. 6 I, IV and VII, it is seen that the largest differences between the filtered series are observed for t > 100. Fig. 6V, VII and 7II show that, in that region, model B and C gives similar representations of the system, while the limitations of model A clearly appear in fig. 6III and 7I. The most important feature is the behaviour of the representation at year 9. For the three models, the distance with the original value is important at that point (about 50, see fig. 6III, 6VI and 6 IX), but the models show different reactions. Model A is not able to adapt itself within a year, which leads to a much larger discrepancy at year 10. Model B and C both reacts to the large error but, only model B succeeds in reducing the error.

On basis of the results reported in this section, it can be concluded that model C gives the most accurate representation of the original series. Therefore, only model C will be considered in the following, when discussing the distance to the yearly results in more details.

Fig. 8. Relative errors (in absolute value) for the yearly and monthly data. The errors are computed by subtracting the original data from the filtered series obtained using model C.

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In fig. 8, the relative differences (in absolute value) are presented for both the yearly (fig. 8I) and monthly (fig. 8II) data. From fig. 8I, it appears that the largest yearly discrepancies occur after the second turning point, with a maximum of 4 %. The repercussions of these errors on the monthly data are reported in fig. 8II, where the maximum error is a factor 2 larger.

Adding to the system the restriction that the discrepancies for the yearly results should be equal to zero[11], brings the problem of distributing the error among the months. The distribution has to be done carefully because it may have some significant repercussion on the accuracy of the representation.

Fig. 9. Graphical representation of the filtered series and their residuals (see text) after correction. Figures I and IV corresponds to a distribution of the yearly discrepancy in equal amount among the months, while figures II and V corresponds to a weighted distribution. Figures III and VI are obtained after applying Denton's method (see text).

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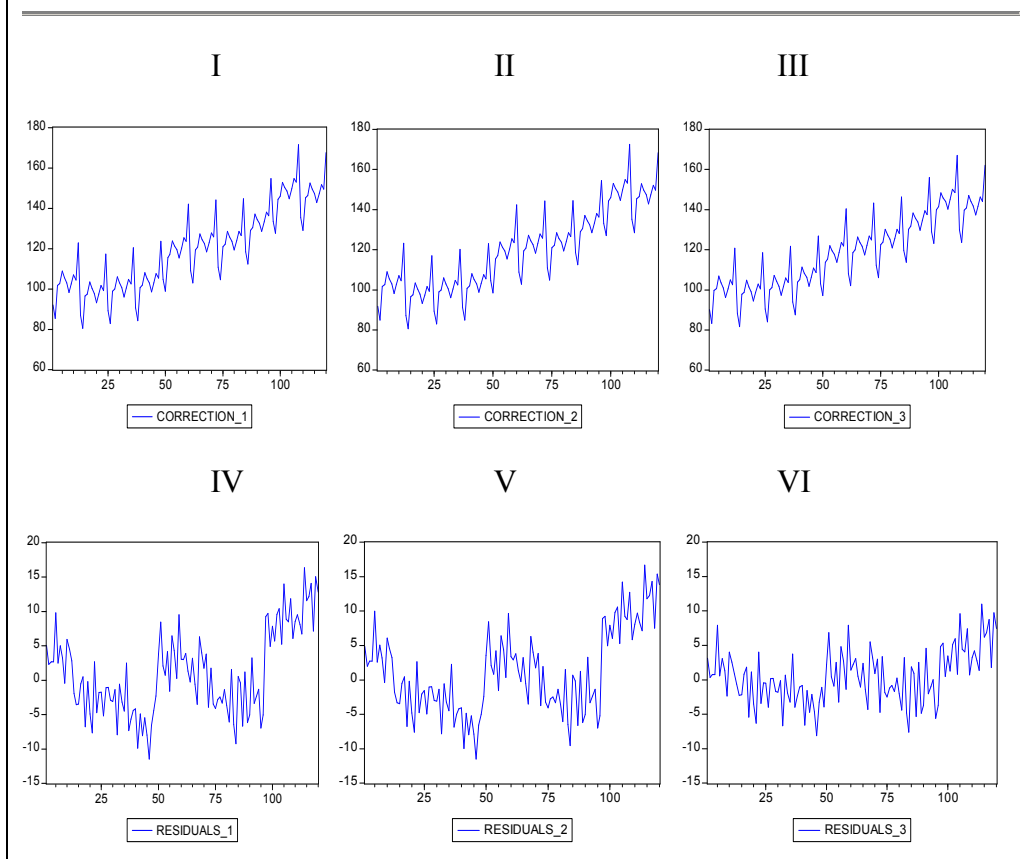
[11] The inclusion of initial conditions in the transition equation, which is needed for representing accurately the data associated with the smallest t values (i.e. at the beginning of the time series), perturbs the representation in such a way that the sum over the months can differ significantly from the yearly data.

In the following, three distribution schemes are investigated. In the first one, the difference is equally distributed over the 12 months. This approach has the advantage that it does not destroy the seasonal pattern but it could bring some year to year shifts in the representation. The second method is based on a proportional distribution of the error. Here, in addition to a possible year to year shift, the seasonal component is altered. A third approach, based on Denton's method (Denton 1971) is also reported. In this approach, the distribution of the yearly discrepancy among the higher frequency periods is done by minimizing a so-called penalty function. In the present case, the penalty function is based on the differences between the first differences of the original and adjusted series (cf. equation (8)). The results for the three correction schemes are reported in fig. 9.

From fig. 9I, II and III, it is not easy to see the differences between the corrected series. The differences become visible when looking at the residuals (fig. 9IV, V and VI). The residuals are computed as the difference between the corrected values and the original series. From fig. 9IV and V, it is seen that the differences between the two first distribution schemes are small. In the two cases the years are considered independently of each other and the two clear "discontinuities" are a typical signature of this effect. On the other hand, the residuals reported in fig. 9VI, show a much smoother behaviour, without any jump.

Comparing the corrected with the non-corrected (i.e. comparing fig. 9I and II with fig. 6VIII), it is seen that the two first series are less accurately described. The inclusion of the restrictions increases the mean amplitude of the residuals. This is well exemplified by the increase of the maximum and minimum error from 10 and -8 to, respectively, 15 and -11. When Denton's method is used, the structure and the amplitude of the residuals are only slightly perturbed and the accuracy of the corrected series is comparable to these of the non-corrected representation.

4. Application

The monthly statistics on retail sales (see section 3.2) that are used among others to estimate the monthly growth of the household consumption are represented in the form of an index series, 1995 being the reference year. The motivation for this is that the statistics do not give a really accurate picture of the sales but it gives a good representation of the relative sales (i.e. the growth). On the other hand, the yearly statistics on retail sales gives an accurate estimation of the money earned within the retail sector. If one wants to

estimate the absolute monthly data, the method presented previously in this paper offers a nice possibility.

Information about the yearly sales from the retail sector is partly available on the website of Statistics Netherlands[12]. Because the accuracy of the state space description improves with t, it was chosen to focus on the first 6 years (i.e. data from 1994 to 1999) for which data are easily available. The description is expected to be sensitive to the initial conditions and the effect of the data adjustment, using Denton's method, can have important repercussions on the accuracy of the representation. If this series is correctly described, it is expected that no major problem will be encountered for the next years (i.e. 2000, 2001 and 2002).

Fig. 10. Original and corrected monthly series built on data from the yearly retail sales statistics (fig. I). In fig. II, the differences between the two series are plotted[13]. The data on the y-axis are in 10^9 €, while on the x-axis the months are numbered from 1 to 72.
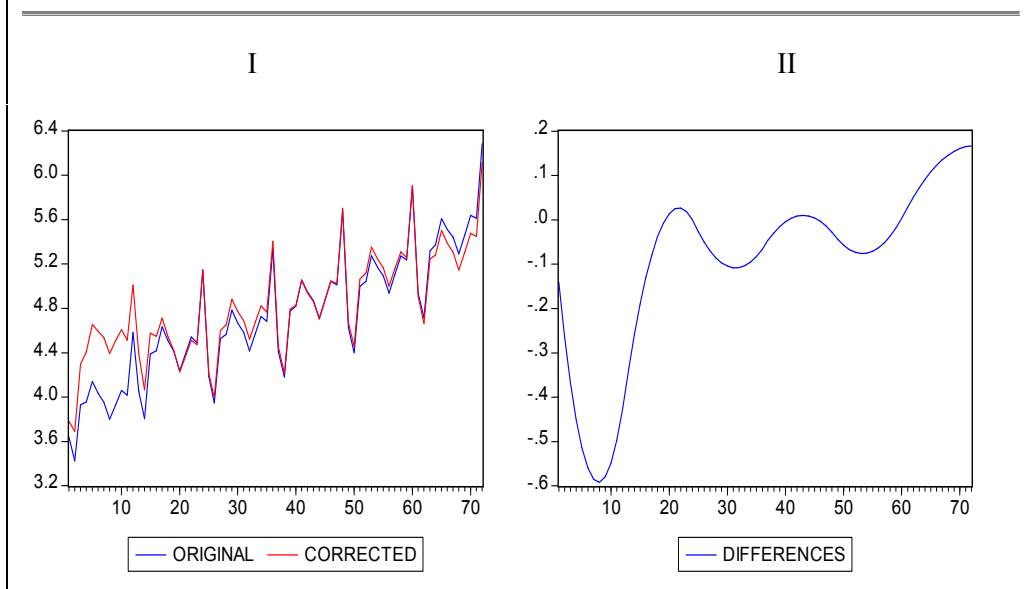
In a first step, the state space approach is applied on the yearly data (reported in table 3, in the 4th, 7th and 10th columns) to obtain a monthly series, called the original series. The seasonal component of this series is computed on basis of the seasonal pattern defined in section 3.2. Then Denton's method is applied to distribute the remaining discrepancies. The results for the original and the corrected series are reported in table 3 and plotted in fig. 10. From

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[12] www.statline.nl
[13] Fig. 10II nicely illustrates how smoothly the discrepancies are distributed when using Denton's method.

the table it appears that applying Denton's algorithm brings the sum of the monthly data in perfect agreement with the values from the yearly statistics.

On basis of these results, it is seen that the differences between the original and the corrected series are large for the smallest and the largest t values (i.e. at the beginning and the end of the time series).

Table 3. Original (Org) and corrected (Cor) series for the monthly retail sales. In addition, the data of the yearly statistics (YS) are reported for comparison. The values are given in 10^9 €.

Month	1994			1995			1996		
	Org	Cor	YS	Org	Cor	YS	Org	Cor	YS
January	3.87	3.94		4.08	4.27		4.21	4.27	
February	3.69	3.81		3.88	4.04		4.02	4.09	
March	4.13	4.31		4.36	4.48		4.50	4.58	
April	4.18	4.40		4.41	4.51		4.56	4.64	
May	4.34	4.60		4.59	4.67		4.74	4.83	
June	4.29	4.57		4.54	4.60		4.70	4.79	
July	4.22	4.52		4.46	4.51		4.63	4.72	
Augustus	4.04	4.34		4.27	4.31		4.44	4.54	
September	4.20	4.49		4.43	4.47		4.62	4.71	
October	4.30	4.59		4.55	4.58		4.73	4.82	
November	4.20	4.46		4.43	4.47		4.63	4.71	
December	4.75	4.98		5.03	5.08		5.23	5.30	
Total	50.23	53.00	53	53.02	54.00	54	55.00	56.00	56

Table 3. Continued.

	1997			1998			1999		
Month	Org	Cor	YS	Org	Cor	YS	Org	Cor	YS
January	4.44	4.50		4.67	4.73		4.97	4.94	
February	4.26	4.31		4.50	4.56		4.81	4.75	
March	4.73	4.77		4.97	5.04		5.28	5.21	
April	4.79	4.83		5.05	5.11		5.36	5.27	
May	4.97	5.01		5.24	5.30		5.56	5.45	
June	4.94	4.97		5.20	5.26		5.53	5.41	
July	4.88	4.91		5.15	5.20		5.48	5.35	
Augustus	4.71	4.74		4.98	5.03		5.32	5.18	
September	4.88	4.92		5.16	5.19		5.50	5.36	
October	5.00	5.04		5.29	5.31		5.63	5.49	
November	4.91	4.95		5.20	5.20		5.55	5.40	
December	5.49	5.54		5.79	5.77		6.14	5.99	
Total	58.00	58.50	58.5	61.20	61.70	61.7	65.14	63.80	63.8

However, neither table 3 nor fig.10 gives any indication about the accuracy of the original and corrected fits. This problem is solved by rescaling the three series. The rescaling is done by dividing the three series by their value for December 1997. The year 1997 was chosen because, during the whole year, the differences between the original and corrected series are small and nearly constant. The choice of the month, December, is arbitrary[14]. The three rescaled series are plotted in fig. 11I. The differences between the monthly data and the original series and between the original data and the corrected series are also plotted (see fig. 11II and III). From these two figures, it is seen that the correction considerably improves the description of the series, especially in the region $1 \leq t \leq 12$ (i.e. the first year) where the error is reduced by about a factor of 2.

Fig. 11. Comparison between the data from the monthly retail sales statistics (DHSALES) and the original and corrected series built from the data of the yearly statistics. In fig. I the three rescaled series are reported (see text), while fig. II and III represent the discrepancies between the series reported in fig. I. Fig. II and III are associated with, respectively, the original and the corrected series. A larger version of fig. I is presented in appendix 1.
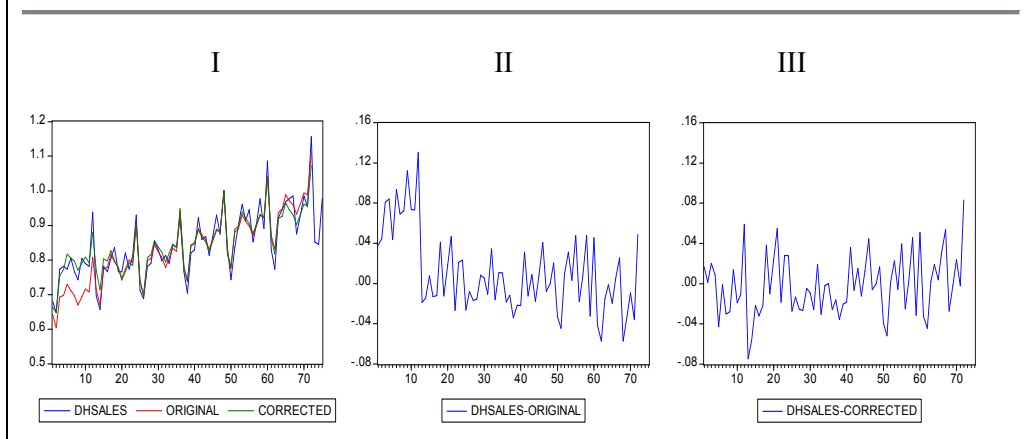
It is worth mentioning that the jump at $t = 12$, although less visible in fig. 11III, still remains. In the two cases, the difference between the residuals at $t = 12$ and $t = 13$ is close to 0.15. Fig. 11II also shows that the original series gives a better representation of the last year (i.e. $60 \leq t \leq 72$). This is well

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[14] The index series uses the data from 1995 as reference for the levels and it could be tempting to use the same year for rescaling the two series. However this is not the most appropriate solution because the accuracy of the method is lower at the beginning of the series which could have some influence on the representation of the year 1995.

exemplified by the value at t = 72 for which the residuals of the original and corrected series are, respectively, 0.05 and 0.08.

5. Conclusions

In the present paper, the state space approach is first tested on two case studies in order to investigate the accuracy of the temporal disaggregation. From the first case, it is seen that Monte-Carlo simulations (based on 2500 runs) are able to retrieve the original DGP, a combination of an AR(1) process and a trend, with a good accuracy. No more than 6 observations are needed to reproduce the coefficient of the slope within less than 1%, while increasing further the number of observations leads to a more accurate representation of the slope, and, more important, to a much smaller standard deviation.

The description of the AR(1) process is shown to be more sensitive to the number of observations. With 25 observations or less, the means of the different Monte-Carlo simulations are not localised close to the DGP value. Using 50 observations or more leads to a much better description of the process but the standard deviation is found to be rather large. This means that basing a representation on a single run can lead to an erroneous description of the short-time behaviour.

In most cases, we are not interested in retrieving the DGP but in describing accurately the disaggregated series. It is shown in the first test case that simulating the series is not the best option because the short-term fluctuations can be poorly represented. The use of the series built with the Kalman filter gives better results. In that case, it is seen that, as expected from the theory, the accuracy of the description is increasing with t, which involves that the last years are better described than the first ones.

The results of the monthly retail sales statistics of the last 10 years are used for the second test case. On basis of the original data, the yearly series is built and the seasonal pattern is extracted. Here, the goal of the study is to investigate whether the AR(1) term is able to compensate for an artificially created lack of information. Three different models are tested and from the analysis of the results, it appears that using a stochastic trend gives enough flexibility to compensate for the lack of information. In that case, the AR(1) term becomes irrelevant.

However, even the best results do not reproduce exactly the yearly data. The largest differences are close to 4 % while, when considering the monthly data, the largest errors are close to 10 %. It is therefore clear that imposing a zero discrepancy between the yearly observations and their state space representations can have significant repercussions on the description of the series.

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The distribution of the discrepancy among the monthly data is investigated by applying 3 different methods. The first two are based on a weighted distribution within the year, while the third one makes use of Denton's algorithm. From the results, it is seen that Denton's approach gives by far the most consequent representation.

The application of these two approaches to the data from the yearly retail sales statistics is shown to give very satisfying results. Because the monthly statistics only report relative levels, the use of yearly data is needed to estimate the absolute monthly sales of the retail sector. The sensitivity of the state space method to the initial condition is responsible for some discrepancy between the sum of the monthly and the yearly data, the largest difference being 2.8 billion euros (~ 5% of the total sales). It is shown that using Denton's approach not only removes all discrepancies but, in addition, it shifts the fitted series in such a way that its shape is closer to the shape of the series built on the data from the monthly statistics.

As a global conclusion, it can be said that the combination of the state space approach with Denton's method provides a powerful tool for performing accurate temporal disaggregation.

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Appendix 1: Enlargement of fig. 11I.